FYJC – MATHEMATICS & STATISTICS

HIGHLIGHTS

✓ Solution to all questions

✓ solutions are put in way the student is expected to reproduce in the exam

✓ taught in the class room the same way as the solution are put up here . That makes the student to easily go through the solution & prepare him/herself when he/she sits back to revise and recall the topic at any given point of time .

✓ lastly, if student due to some unavoidable reasons , has missed the lecture , will not have to run here and there to update his/her notes .

✓ however class room lectures are must for easy passage of understanding & learning the minuest details of the given topic

PAPER – I

EQUATIONS

Compiled & Conducted @ JKSC

01. The ages of two persons A and B are in the ratio 7:5 . Ten years hence , the ratio of their ages will be 9:7 . Find their ages

    ans : 35 , 25

02. Kiran has 85 currency notes in all , some of which were of ₹ 100 denomination and the remaining of ₹ 50 denomination . The total amount of all these currency notes was ₹ 5,000 . How much amount did she have in the denomination of ₹ 50

    ans : 3500

03. Two articles A and B together cost ₹ 2,226 . If article A cost 10% more than B find the cost of each article .

    ans : 1,166 , 1060

04. The profit earned by selling a machine for ₹ 900 is double the loss incurred when the same machine is sold for ₹ 450 . At what price should the machine be sold to make 25% profit

    ans : 750

05. The profit earned by selling an article for ₹ 832 is equal to loss incurred when the same article is sold for ₹ 448 . What should be the selling price for making 50% profit

    ans : 960

06. Akbar divides Rs 8600 among 5 sons , 4 daughters and 2 nephews . If each daughter receives 4 times as much as each nephew and each son receives 5 times as much as each nephew , how much does each daughter receive

    ans : 800

07. the difference between the compound interest and simple interest on a certain sum at 10% p.a. for 2 years is ₹ 631 . find the sum

    ans : 63,100

08. In a class , 9 students remain standing if each student occupies one seat . If two students occupy one seat , 7 seats are left empty . Find the number of seats in the class and the number of students in it

    ans : 23 , 32

09. The speed of a boat in still water is 8 km/hr . If it takes the same time in going 20km downstream as it takes in going 12km upstream , find the speed of the water of the river .

    ans : 2

01. The ages of two persons A and B are in the ratio 7:5 . Ten years hence , the ratio of their ages will be 9:7 . Find their ages

SOLUTION :

let A's age = 7x

B's age = 5x

10 years hence ;

A's age = 7x + 10

B's age = 5x + 10

As per the given condition ;

$$\frac{7x + 10}{5x + 10} = \frac{9}{7}$$

$$49x + 70 = 45x + 90$$

$$4x = 20$$

$$x = 5$$

A's age = 7x = 35 yrs

B's age = 5x = 25 yrs

02. Kiran has 85 currency notes in all , some of which were of ₹ 100 denomination and the remaining of ₹ 50 denomination . The total amount of all these currency notes was ₹ 5,000 . How much amount did she have in the denomination of ₹ 50

SOLUTION

Let 50 rupee notes = x

100 rupee notes = 85 – x

The total value = 5000

$$50x + 100(85 - x) = 5000$$

$$50x + 8500 - 100x = 5000$$

$$8500 - 50x = 5000$$

$$3500 = 50x$$

$$x = 70$$

Hence ;

50 rupee notes = 70

value = Rs 3,500

03. Two articles A and B together cost ₹ 2,226 . If article A cost 10% more than B find the cost of each article

SOLUTION

Let cost of article B = x

∴ cost of article A = $x + \dfrac{10\,x}{100}$

$$= x + \frac{x}{10}$$

$$= \frac{11x}{10}$$

As per the given condition ;

$$x + \frac{11x}{10} = 2226$$

$$\frac{21x}{10} = 2226$$

$$x = 1060$$

Hence cost of article B = Rs 1060

Cost of article A = $\dfrac{11(1060)}{10}$

$$= Rs\ 1,166$$

04. The profit earned by selling a machine for ₹ 900 is double the loss incurred when the same machine is sold for ₹ 450 . At what price should the machine be sold to make 25% profit

SOLUTION

let CP = x

On selling at Rs 900 ; Profit = 900 – x

On selling at Rs 450 ; Loss = x – 450

As per the given condition

$$900 - x = 2(x - 450)$$

$$900 - x = 2x - 900$$

$$1800 = 3x$$

$$x = 600$$

Hence

CP = 600

Add 25% profit

SP = $600 + \dfrac{25}{100}(600)$

$$= Rs\ 750$$

05. The profit earned by selling an article for ₹ 832 is equal to loss incurred when the same article is sold for ₹ 448 . What should be the selling price for making 50% profit

SOLUTION

let CP = x

On selling at Rs 832 ;   Profit = 832 – x

On selling at Rs 448 ;   Loss   = x – 448

As per the given condition

$$832 - x = x - 448$$

$$1280 = 2x$$

$$x = 640$$

Hence

CP   = 640

Add 50% profit

$$SP = 640 + \frac{50}{100}(640)$$

$$= 640 + 320$$

$$= Rs\ 960$$

06. Akbar divides ₹ 8600 among 5 sons , 4 daughters and 2 nephews . If each daughter receives 4 times as much as each nephew and each son receives 5 times as much as each nephew , how much does each daughter receive

SOLUTION

let nephew receive = x

∴ daughter receives = 4x

son receives   = 5x

Akbar divides Rs 8600 among 5 sons , 4 daughters and 2 nephews

$$5(5x) + 4(4x) + 2(x) = 8600$$

$$43x = 8600$$

$$x = 200$$

∴ Daughter receives = 4(200)

$$= Rs\ 800$$

07. the difference between the compound interest and simple interest on a certain sum at 10% p.a. for 2 years is ₹ 631 . find the sum

SOLUTION

let the principal amount (sum)   = x

$$SI = x \cdot 2 \cdot \frac{10}{100} = \frac{x}{5}$$

$$CI = x\left(1 + \frac{10}{100}\right)^2 - x$$

$$= x\left(\frac{11}{10}\right)^2 - x$$

$$= \frac{121}{100}x - x \qquad = \frac{21x}{100}$$

Given : CI – SI = 631

$$\frac{21x}{100} - \frac{x}{5} = 631$$

$$\frac{21x - 20x}{100} = 631$$

$$\frac{x}{100} = 631$$

$$x = Rs\ 63,100$$

08. In a class , 9 students remain standing if each student occupies one seat . If two students occupy one seat , 7 seats are left empty . Find the number of seats in the class and the number of students in it

SOLUTION

let number of seats in the class = x

according to condition 1 : 9 students remain standing if each student occupies one seat

total students = x + 9

according to condition 2 : 7 seats are left empty if 2 students occupy one seat

total students = 2(x – 7)

$$x + 9 = 2(x - 7)$$

$$x + 9 = 2x - 14$$

$$23 = x$$

no. of seats in the class = 23

no. of students = 23 + 9 = 32

09. The speed of a boat in still water is 8 km/hr . If it takes the same time in going 20km downstream as it takes in going 12km upstream , find the speed of the water of the river

Let

speed of the river water = x km/hr

speed of the boat in still water = 8

speed of boat during downstream
$$= 8 + x \text{ km/hr}$$
speed of boat during upstream
$$= 8 - x \text{ km/hr}$$

time taken in going 20 km down stream
$$= 20/(8 + x)$$

time taken in going 12 km upstream
$$= 12/(8 - x)$$

according to given condition :

$$\frac{20}{8 + x} = \frac{12}{8 - x}$$

$$160 - 20x = 96 + 12x$$

$$64 = 32x$$

$$x = 2$$

speed of the river water = 2 km/hr

EQUATIONS – EX 8.2

01. Hariram buys 4 cell phones and 9 calculators for ₹ 13,400 . If he sells the cell phones at 10% profit and the calculators at 20% profit , then he earns a total profit of ₹ 1880 . Find the cost of a cellphone and a calculator

02. two numbers are such that 4 times the smaller one is less than 3 times the larger one by 5 . If the sum of the numbers is larger than 6 times their difference by 6 , find the two numbers

03. there are two examination rooms A and B . If 10 examinees are sent from A to B then the number of examinees in each room is same If 20 examinees are sent from B to A , then the number of examinees in A is double the number of examinees in B . Find the number of examinees in each room

04. A and B , each has a certain number of chocolates . A says to B , " if you give me 30 chocolates , I shall have twice as many as left with you" . B replies ' " if you give me 10 chocolates , I shall have thrice as many as left with you" How many chocolates does A and B have

05. the sum of numerator and denominator of a fraction is 11 . If 1 is added to the numerator and 2 is subtracted from the denominator , it becomes 2/3 . Find the fraction .

06. three cricket players Suresh , Azhar and Venkat participated in a cricket match find their individual score of runs x , y , z if

    i) the sum of their scores is a century

    ii) if the sum of scores of Suresh and Azhar is subtracted from twice of score of Venkat , it is a half century

    iii) subtraction of score of Suresh from 4 times score of Azhar is equal to the score of Venkat

01. Hariram buys 4 cell phones and 9 calculators for ₹ 13,400 . If he sells the cell phones at 10% profit and the calculators at 20% profit , then he earns a total profit of ₹ 1880 . Find the cost of a cellphone and a calculator

SOLUTION :

cost of one cell phone $= x$

cost of one calculator $= y$

Hariram buys 4 cell phones and 9 calculators for ₹ 13,400

$4x + 9y = 13400$ .... (1)

total profit of ₹ 1880 on selling cell phones at 10% profit and the calculators at 20% profit

$$\frac{10}{100}(4x) + \frac{20}{100}(9y) = 1880$$

$4x + 18y = 18800$ .... (2)

(2) − (1) $9y = 5400$

$y = 600$

subs in 1: $x = 2000$

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02. two numbers are such that 4 times the smaller one is less than 3 times the larger one by 5 . If the sum of the numbers is larger than 6 times their difference by 6 , find the two numbers

SOLUTION :

let smaller number $= x$

larger number $= y$

according to condition 1 :

$4x = 3y − 5$

$4x − 3y = −5$ .... (1)

according to condition 1 :

$x + y = 6(y − x) + 6$

$x + y = 6y − 6x + 6$

$7x − 5y = + 6$ ....(2)

(1) x 5 $20x − 15y = −25$

(2) x 3 $21x − 15y = +18$

$x = 43$

subs in (1) $y = 39$

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03. there are two examination rooms A and B . If 10 examinees are sent from A to B then the number of examinees in each room is same If 20 examinees are sent from B to A , then the number of examinees in A is double the number of examinees in B . Find the number of examinees in each room .

SOLUTION :

let number of examinees in A $= x$

number of examinees in B $= y$

according to condition 1

$x − 10 = y + 10$

$x − y = 20$ .... (1)

according to condition 2

$x + 20 = 2(y − 20)$

$x + 20 = 2y − 40$

$x − 2y = − 60$ .... (2)

(1) − (2) : $y = 40$

$x = 60$

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04. A and B , each has a certain number of chocolates . A says to B , " if you give me 30 chocolates , I shall have twice as many as left with you" . B replies ' " if you give me 10 chocolates , I shall have thrice as many as left with you" How many chocolates does A and B have

SOLUTION :

let   A have $= x$ chocolates

B have $= y$ chocolates

According to condition 1 :

$x + 30 = 2(y − 30)$

$x + 30 = 2y − 60$

$x − 2y = − 90$ .... (1)

According to condition 2 :

$y + 10 = 3(x − 10)$

$y + 10 = 3x − 30$

$3x − y = 40$ .... (2)

(1) x 3 : $3x − 6y = −270$

(2) $3x − y = 40$

$5y = 310$

$y = 62$

subs in (1) $x = 34$

05. the sum of numerator & denominator of fraction is 11 . If 1 is added to numerator & 2 is subtracted from denominator , it becomes 2/3 . Find the fraction .

SOLUTION :

let   fraction   = x/y

according to condition 1 :

$$x + y = 11 \quad \text{.......} \quad (1)$$

according to condition 2

$$\frac{x + 1}{y - 2} = \frac{2}{3}$$

$$3x + 3 = 2y - 4$$

$$3x - 2y = -7 \quad \text{........} \quad (2)$$

(1) x 2   : $2x + 2y = 22$

$$\underline{\ 3x - 2y = -7}$$

$$5x \quad\quad = 15$$

$$x \quad\quad = 3$$

subs in (1) $y \quad = 8$

original fraction   = 3/8

06. three cricket players Suresh , Azhar and Venkat participated in a cricket match find their individual score of runs x , y , z

i) the sum of their scores is a century

ii) if the sum of scores of Suresh and Azhar is subtracted from twice of score of Venkat , it is a half century

iii) subtraction of score of Suresh from 4 times score of Azhar is equal to the score of Venkat

SOLUTION :

$$x + y + z = 100 \quad \text{.......} \quad (1)$$

$$2z - (x + y) = 50 \quad \text{......} \quad (2)$$

$$4y - x = z \quad \text{.......} \quad (3)$$

subs (3) in (1) :

$$x + y + 4y - x = 100$$

$$5y = 100 \quad \therefore \ y = 20$$

subs (3) in (2) :

$$8y - 2x - x - y = 50$$

$$7y - 3x = 50$$

$$140 - 3x = 50$$

$$140 - 50 = 3x \quad \therefore \ x = 30$$

subs x = 30 & y = 20 in (1)   z = 50

6

01. the number 50 is divided into two parts such that sum of their reciprocals is 1/12. find the two parts

02. for a two digit number , the tens digit is bigger . The product of the digits is 27 and difference between two digits is 6 . Find the number

03. the hypotenuse of a right angled triangle is 26 cm and sum of other two sides is 34 cm . Find the two sides

04. the age of father is twice the square of the age of his son . Eight years hence , the age of father will be 4 years more than the three times the age of son . Find their present age

05. A footpath of uniform width runs around the inside of a rectangular garden 32 m long and 24m wide . If path occupies 208 sq m , find the width of the footpath

06. Rita visits her friends house which is at a distance of 12 km . She covers half of the distance at a speed of x km/hr snd the remaining distance at a speed of (x + 2) km/hr . If she takes 2 hrs 30 min to cover the whole distance , find x

07. a plane left 30 minutes late than the scheduled time and in order to reach its destination 1500 km away in time , it had to increase its speed by 250 km/hr from its usual speed . Find its usual speed .

08. a certain number of tennis balls were purchased for ₹ 450 . Five more balls could have been purchased for the same amount if each ball was cheaper by ₹ 15 .Find the number of balls purchased

09. a piece of ribbon cost ₹ 35 . If the length of the piece would have been 4m longer and each meter cost ₹ 1 less , the cost of the ribbon would have remained unchanged . What is the length of the piece of ribbon

10. a number of friends decided to go for a picnic and planned to spend ₹ 96 on chocolates . Four of them , however did not turn up. As a consequence , the remaining ones had to contribute Rs 4 each extra . Find the number of friends who attended the picnic

11. the total cost price of a certain numer of identical articles is ₹ 4800 . By selling the articles at Rs 100 each , a profit equal to cost price of 15 articles is gained . find the number of articles

01. the number 50 is divided into two parts such that sum of their reciprocals is 1/12. find the two parts

SOLUTION :

let the two parts be x & 50 – x
according to given condition :

$$\frac{1}{x} + \frac{1}{50 - x} = \frac{1}{12}$$

$$\frac{50 - x + x}{x(50 - x)} = \frac{1}{12}$$

$$600 = 50x - x^2$$

$$x^2 - 50x + 600 = 0$$

$$(x - 30)(x - 20) = 0$$

x = 30 OR x = 20 ,   two parts :   20 , 30

02. for a two digit number , the tens digit is bigger . The product of the digits is 27 and difference between two digits is 6 . Find the number

SOLUTION :

let the digit in unit place   = x
∴ the digit in tens place   = x + 6

Given : Product of digits = 27

$$x(x + 6) = 27$$

$$x^2 + 6x - 27 = 0$$

$$(x + 9)(x - 3) = 0 \qquad x = 3$$

number = 93

03. the hypotenuse of a right angled triangle is 26 cm and sum of other two sides is 34 cm . Find the two sides

SOLUTION :

let one side = x
∴ other side = 34 – x

By PYTHAGORAS THEOREM

$$x^2 + (34 - x)^2 = 26^2$$

$$x^2 + 1156 - 68x + x^2 = 676$$

$$2x^2 - 68x + 480 = 0$$

$$x^2 - 34x + 240 = 0$$

$$(x - 10)(x - 24) = 0$$

x = 10   , x = 24
the two sides are   :   10 , 24

04. the age of father is twice the square of the age of his son . Eight years hence , the age of father will be 4 years more than the three times the age of son . Find their present age

SOLUTION :

let present age of son = x
∴ present age of father = $2x^2$

Eight years hence ;
Fathers age   = $2x^2 + 8$
Sons age   = x + 8

According to given condition ;
$$2x^2 + 8 = 3(x + 8) + 4$$

$$2x^2 + 8 = 3x + 24 + 4$$

$$2x^2 - 3x - 20 = 0$$

$$2x^2 - 8x + 5x - 20 = 0$$

$$2x(x - 4) + 5(x - 4) = 0$$

$$(2x + 5)(x - 4) = 0$$

x = 4
Present age of son = 4 yrs
Present age of father = $2(4)^2$ = 32 yrs

05. A footpath of uniform width runs around the inside of a rectangular garden 32 m long and 24m wide . If path occupies 208 sq m , find the width of the footpath

SOLUTION :

let width of footpath = x m

area of foot path
= area of outer rectangle – area of inner rectangle

$$208 = 32 \times 24 - (32 - 2x)(24 - 2x)$$

$$208 = 768 - (768 - 64x - 48x + 4x^2)$$

$$208 = 768 - (768 - 112x + 4x^2)$$

$$208 = 112x - 4x^2$$

$$4x^2 - 112x + 208 = 0$$

$$x^2 - 28x + 52 = 0$$

$$(x - 26)(x - 2) = 0$$

x ≠ 26 (width of footpath cannot be more than width of garden)

x = 2

06. Rita visits her friends house which is at a distance of 12 km . She covers half of the distance at a speed of x km/hr snd the remaining distance at a speed of (x + 2) km/hr . If she takes 2 hrs 30 min to cover the whole distance , find x

SOLUTION :

$$\frac{6}{x} + \frac{6}{x+2} = \frac{5}{2}$$

$$6\left(\frac{1}{x} + \frac{1}{x+2}\right) = \frac{5}{2}$$

$$6\left(\frac{x + 2 + x}{x(x+2)}\right) = \frac{5}{2}$$

$$12(2x + 2) = 5x(x + 2)$$

$$24x + 24 = 5x^2 + 10x$$

$$5x^2 - 14x - 24 = 0$$

$$5x^2 - 20x + 6x - 24 = 0$$

$$5x(x - 4) + 6(x - 4) = 0$$

$$(5x + 6)(x - 4) = 0$$

$$x = 4 \text{ km/hr}$$

07. a plane left 30 minutes late than the scheduled time and in order to reach its destination 1500 km away in time , it had to increase its speed by 250 km/hr from its usual speed . Find its usual speed .

SOLUTION :

Let the usual speed = x km/hr

According to given condition
(plane due to extra speed covers same distance of 1500 km in ½ hr less time)

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$$1500\left(\frac{1}{x} - \frac{1}{x+250}\right) = \frac{1}{2}$$

$$1500\left(\frac{250}{x(x + 250)}\right) = \frac{1}{2}$$

$$2(1500)250 = x^2 + 250x$$

$$x^2 + 250x - 750000 = 0$$

$$(x + 1000)(x - 750) = 0$$

$$x = 750 \text{ km/hr}$$

08. a certain number of tennis balls were purchased for ₹ 450 . Five more balls could have been purchased for the same amount if each ball was cheaper by ₹ 15 .Find the number of balls purchased

SOLUTION :

let number of balls = x
price of 1 ball = $\frac{450}{x}$

Five more balls could have been purchased for the same amount if each ball was cheaper by ₹ 15

$$\left(\frac{450}{x} - 15\right) . (x + 5) = 450$$

$$(450 - 15x) . (x + 5) = 450x$$

$$450x + 2250 - 15x^2 - 75x = 450x$$

$$2250 - 15x^2 - 75x = 0$$

$$15x^2 + 75x - 2250 = 0$$

$$x^2 + 5x - 150 = 0$$

$$(x + 15)(x - 10) = 0$$

$$x = 10$$

09. a piece of ribbon cost ₹ 35 . If the length of the piece would have been 4m longer and each meter cost ₹ 1 less , the cost of the ribbon would have remained unchanged . What is the length of the piece of ribbon .

SOLUTION :

let length of ribbon = x meters
cost of 1 meter = 35/x

If the length of the piece would have been 4m longer and each meter cost Rs 1 less , the cost of the ribbon would have remained unchanged

$$(x + 4)\left(\frac{35}{x} - 1\right) = 35$$

$$(x + 4) . (35 - x) = 35x$$

$$35x - x^2 + 140 - 4x = 35x$$

$$-x^2 + 140 - 4x = 0$$

$$x^2 + 4x - 140 = 0$$

$$(x + 14)(x - 10) = 0 \qquad x = 10$$

10. a number of friends decided to go for a picnic and planned to spend ₹ 96 on chocolates . Four of them , however did not turn up. As a consequence , the remaining ones had to contribute ₹ 4 each extra . Find the number of friends who attended the picnic

SOLUTION :

let number of friends that decided to go for a picnic = x

∴ contribution / friend = 96/x

as four did not turn up , remaining ones had to contribute Rs 4 extra

$$(x - 4) \cdot \left(\frac{96}{x} + 4 \right) = 96$$

$$(x - 4)(96 + 4x) = 96x$$

$$96x + 4x^2 - 384 - 16x = 96x$$

$$4x^2 - 16x - 384 = 0$$

$$x^2 - 4x - 96 = 0$$

$$(x - 12)(x + 8) = 0 \qquad x = 12$$

Hence no of friends who attended the picnic = 12 − 4 = 8

11. the total cost price of a certain numer of identical articles is ₹ 4800 . By selling the articles at ₹ 100 each , a profit equal to cost price of 15 articles is gained . find the number of articles .

SOLUTION :

let number of articles = x

total cost price = 4800

∴ cost of one article = $\dfrac{4800}{x}$

each article is sold at Rs 100

total selling price = 100x

given : a profit equal to cost price of 15 articles

$$100x - 4800 = 15\left(\frac{4800}{x} \right)$$

$$x - 48 = \frac{15(48)}{x}$$

$$x^2 - 48x = 720$$

$$x^2 - 48x - 720 = 0$$

$$(x - 60)(x + 12) = 0$$

$$x = 60$$

CUBIC EQUATIONS

In General cubic equation is of the form

$$ax^3 + bx^2 + cx + d = 0$$

let α , β , γ be the roots of the above equation

in that case the cubic equation can be arrived at using

$$(x - \alpha) . (x - \beta) . (x - \gamma) = 0$$

$$x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\alpha\gamma)x - \alpha\beta\gamma = 0$$

Comparing it with

$ax^3 + bx^2 + cx + d = 0$ i.e.

$$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$$

we arrive at following results

$\alpha + \beta + \gamma$	$= -\dfrac{b}{a}$
$\alpha\beta + \beta\gamma + \alpha\gamma$	$= \dfrac{c}{a}$
$\alpha\beta\gamma$	$= -\dfrac{d}{a}$

01. find the roots of the cubic equation
$x^3 - 4x^2 + 5x - 2 = 0$

SOLUTION :
since the sum of coefficients on the LHS is zero , x − 1 is a factor of the polynomial

$\therefore$ $x^3 - 4x^2 + 5x - 2 = (x - 1) (\ldots\ldots)$

to find the other factors we use synthetic division

1	1	−4	5	−2
		1	−3	2
	1	−3	2	0

Hence

$x^3 - 4x^2 + 5x + 2 = 0$

$(x - 1)(x^2 - 3x + 2) = 0$

$(x - 1)(x - 1)(x - 2) = 0$

$x = 1 , 1 , 2$

02. Solve the cubic equation
$2x^3 + x^2 - 5x + 2 = 0$

SOLUTION :

since the sum of coefficients on the LHS is zero , x − 1 is a factor of the polynomial

$\therefore$ $2x^3 + x^2 - 5x + 2 = (x - 1) (\ldots\ldots)$

to find the other factors we use synthetic division

1	2	1	−5	2
		2	3	−2
	2	3	−2	0

Hence

$2x^3 + x^2 - 5x + 2 = 0$

$(x - 1)(2x^2 + 3x - 2) = 0$

$(x - 1)(2x^2 + 4x - x - 2) = 0$

$(x - 1).\left[(2x(x + 2) - 1(x + 2)\right] = 0$

$(x - 1)(x + 2)(2x - 1) = 0$

$x = 1 , -2 , 1/2$

02. Solve the cubic equation
$3x^3 - 10x^2 + x + 6 = 0$ given that two of its roots α & β satisfy the relation $\alpha\beta - \alpha - 2 = 0$

SOLUTION :

since the sum of coefficients on the LHS is zero , $x - 1$ is a factor of the polynomial

$\therefore 3x^3 - 10x^2 + x + 6 = (x - 1) (\ldots\ldots)$

to find the other factors we use synthetic division

1	3	−10	1	6
		3	− 7	−6
	3	− 7	− 6	0

Hence

$3x^3 - 10x^2 + x + 6 = 0$

$(x - 1)(3x^2 - 7x - 6) = 0$

$(x - 1)(3x^2 - 9x + 2x - 6) = 0$

$(x - 1). \left[(3x(x - 3) + 2(x - 3)\right] = 0$

$(x - 1)(x - 3)(3x + 2) = 0$

$x = 1 , 3 , {}^{-2}/_3$

03. Solve the equation
$3x^3 - 26x^2 + 52x - 24 = 0$, if its roots are in GP

SOLUTION :

STEP 1 :

$3x^3 - 26x^2 + 52x - 24 = 0$

Comparing with

$ax^3 + bx^2 + cx + d = 0$

$a = 3 , b = -26 , c = 52 , d = -24$

$\alpha\beta\gamma = -\dfrac{d}{a} = \dfrac{24}{3} = 8$

STEP 2 :

Since roots are in GP ,
let roots be : $\dfrac{q}{r}$; q ; qr

$\alpha\beta\gamma = 8$

$q^3 = 8 \quad q = 2$ i.e $\beta = 2$

STEP 3 :
Since one of the roots is 2 ; $x - 2$ is a factor of the polynomial

$\therefore 3x^3 - 26x^2 + 52x - 24 = (x - 2) (\ldots\ldots)$

to find the other factors we use synthetic division

2	3	−26	52	− 24
		6	−40	24
	3	−20	12	0

Hence

$3x^3 - 26x^2 + 52x - 24 = 0$

$(x - 2)(3x^2 - 20x + 12) = 0$

$(x - 2)(3x^2 - 18x - 2x + 12) = 0$

$(x - 2). \left[(3x(x - 6) - 2(x - 6)\right] = 0$

$(x - 2)(x - 6)(3x - 2) = 0$

$x = 2 , 6 , 2/3$

04. Solve the equation
$x^3 - 5x^2 - 16x + 80 = 0$, if its roots are equal in magnitude but opposite in sign

SOLUTION :

STEP 1 :

$x^3 - 5x^2 - 16x + 80 = 0$

Comparing with

$ax^3 + bx^2 + cx + d = 0$

$a = 1 , b = - 5 , c = -16 , d = 80$

$\alpha + \beta + \gamma = -\dfrac{b}{a} = \dfrac{5}{1} = 5$

STEP 2 :

Since roots are equal in magnitude but opposite in sign

$\alpha + \beta = 0$

subs in : $\alpha + \beta + \gamma = 5$

$\gamma = 5$

STEP 3 :

Since one of the roots is 5 ; $x - 5$ is a factor of the polynomial

$\therefore x^3 - 5x^2 - 16x + 80 = (x - 5) (\ldots\ldots)$

to find the other factors we use synthetic division

5	1	− 5	−16	80
		5	0	− 80
	1	0	− 16	0

Hence

$$x^3 - 5x^2 - 16x + 80 = 0$$

$$(x - 5)(x^2 - 16) = 0$$

$$(x - 5)(x - 4)(x + 4) = 0$$

$$x = 5, 4, -4$$

05. Solve the equation
$x^3 - 5x^2 - 2x + 24 = 0$, given that product of its two roots is 12

SOLUTION :

STEP 1 :

$$x^3 - 5x^2 - 2x + 24 = 0$$

Comparing with

$$ax^3 + bx^2 + cx + d = 0$$

$a = 1, b = -5, c = -2, d = 24$

$$\alpha\beta\gamma = -\frac{d}{a} = \frac{-24}{1} = -24$$

STEP 2 :

Since product of roots is 12, $\alpha\beta = 12$

subs in : $\alpha\beta\gamma = -24$

$$12\gamma = -24$$

$$\gamma = -2$$

STEP 3 :

Since one of the roots is -2 ; $x + 2$ is a factor of the polynomial

$\therefore \ x^3 - 5x^2 - 2x + 24 = (x + 2)(\ldots\ldots)$

to find the other factors we use synthetic division

```
-2 |  1   -5   -2    24
   |      -2   14   -24
   ───────────────────────
      1   -7   12     0
```

Hence

$$x^3 - 5x^2 - 2x + 24 = 0$$

$$(x + 2)(x^2 - 7x + 12) = 0$$

$$(x + 2)(x - 3)(x - 4) = 0$$

$$x = -2, 3, 4$$

06. Solve the equation
$2x^3 - 15x^2 + 37x - 30 = 0$, if its roots are in AP

SOLUTION :

STEP 1 :

$$2x^3 - 15x^2 + 37x - 30 = 0$$

Comparing with

$$ax^3 + bx^2 + cx + d = 0$$

$a = 2, b = -15, c = 37, d = -30$

$$\alpha + \beta + \gamma = -\frac{b}{a} = \frac{15}{2}$$

STEP 2 :

Since roots are in AP ,
Let roots be $p - q, p, p + q$

Subs in : $\alpha + \beta + \gamma = \frac{15}{2}$

$$3p = {}^{15}/_2$$

$$p = {}^5/_2$$

$$\therefore \ \beta = {}^5/_2$$

STEP 3 :

Since one of the roots is ${}^5/_2$; $x - {}^5/_2$ is a factor of the polynomial

$\therefore 2x^3 - 15x^2 + 37x - 30 = (x - {}^5/_2)(\ldots\ldots)$

to find the other factors we use synthetic division

```
 5/2 |  2   -15   37   -30
     |        5  -25    30
     ─────────────────────────
        2   -10   12     0
```

Hence

$$3x^3 - 26x^2 + 52x - 24 = 0$$

$$(x - {}^5/_2)(2x^2 - 10x + 12) = 0$$

$$(x - {}^5/_2)(2x^2 - 6x - 4x + 12) = 0$$

$$(x - {}^5/_2).\big[(2x(x - 3) - 4(x - 3)\big] = 0$$

$$(x - {}^5/_2)(x - 3)(2x - 4) = 0$$

$$x = {}^5/_2, 3, 2$$

07. if the roots of the equation
$x^3 - 3x^2 + x + 1 = 0$
are $p - q, p, p + q$, find p & q

SOLUTION :

STEP 1 :

$$x^3 - 3x^2 + x + 1 = 0$$

Comparing with

$$ax^3 + bx^2 + cx + d = 0$$

$a = 1, b = -3, c = 1, d = 1$

$$\alpha + \beta + \gamma = -\frac{b}{a} = 3$$

$$\alpha\beta\gamma = -\frac{d}{a} = -1$$

13

STEP 2 :

Since roots are p – q , p , p + q

Subs in : $\alpha + \beta + \gamma = 3$

$$3p = 3$$
$$p = 1$$

subs in : $\alpha\beta\gamma = -1$

$$(p-q)p(p+q) = -1$$

subs p = 1: $(1-q)1(1+q) = -1$

$$1 - q^2 = -1$$
$$q^2 = 2$$
$$q = \pm\sqrt{2}$$

08. Solve the equation :
$x^3 - 7x^2 + 36 = 0$ given that one root is double the other

SOLUTION :

STEP 1 :

$x^3 - 7x^2 + 36 = 0$

Comparing with

$ax^3 + bx^2 + cx + d = 0$

a = 1 , b = – 7 , c = 0 , d = 36

$\alpha + \beta + \gamma = -\dfrac{b}{a} = 7$

$\alpha\beta + \beta\gamma + \alpha\gamma = \dfrac{c}{a} = 0$

$\alpha\beta\gamma = -\dfrac{d}{a} = -36$

STEP 2 :

Subs $\alpha = 2\beta$ in

$\alpha + \beta + \gamma = 7$

$3\beta + \gamma = 7$

$\gamma = 7 - 3\beta$

Now ,

subs $\alpha = 2\beta$ & $\gamma = 7-3\beta$ in

$\alpha\beta + \beta\gamma + \alpha\gamma = 0$

$2\beta\beta + \beta(7-3\beta) + 2\beta(7-3\beta) = 0$

$2\beta^2 + 7\beta - 3\beta^2 + 14\beta - 6\beta^2 = 0$

$21\beta - 7\beta^2 = 0$

$21\beta = 7\beta^2$

$\beta = 3$

$\alpha = 2\beta = 6$

$\gamma = 7-3\beta = -2$

09. Find the cubic equation with sum , sum of product of its roots taken two at a time and product of its roots as 2 , –7 , –14 respectively

SOLUTION :

Cubic equation can be formed using ,

$x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\alpha\gamma)x - \alpha\beta\gamma = 0$

$x^3 - (2)x^2 + (-7)x - (-14) = 0$

$x^3 - 2x^2 - 7x + 14 = 0$

10. if α , β , γ are the three roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$ such that $\alpha+\beta+\gamma = 5$, $\alpha\beta + \beta\gamma + \alpha\gamma = -16$ and $\alpha\beta\gamma = 80$, find a , b , c , d

SOLUTION :

Cubic equation can be formed using ,

$x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\alpha\gamma)x - \alpha\beta\gamma = 0$

$x^3 - (5)x^2 + (-16)x - 80 = 0$

$x^3 - 5x^2 - 16x - 80 = 0$

Comparing with

$ax^3 + bx^2 + cx + d = 0$

a = 1 , b = – 5 , c = –16 , d = –80